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Analysis of electrostatic force on non-uniformly charged particles

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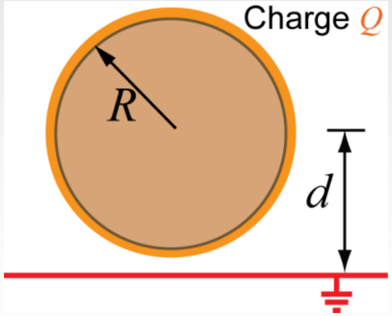
Introduction

- Electrostatic adhesion of charged particles on a substrate
- Simple estimation of adhesion for particle radius R, charge Q at height d

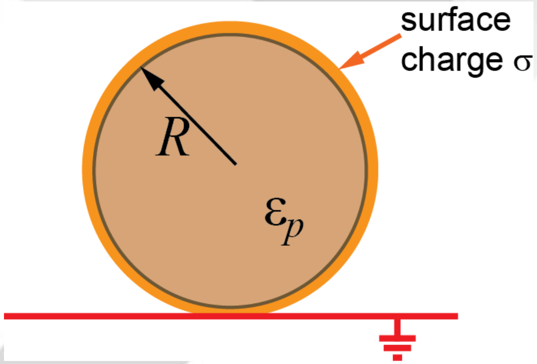
$$F_a = \frac{Q^2}{4\pi\varepsilon_E\varepsilon_0(R+d)^2}$$



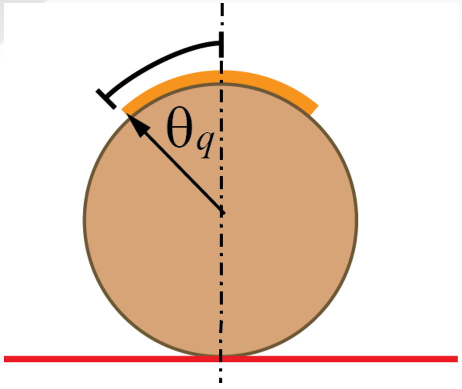
- Estimation is smaller than measured values.
- To explain discrepancy: proximity force, charge patch models
- This work tries to clarify the role of charge distributions on the adhesion

Configuration

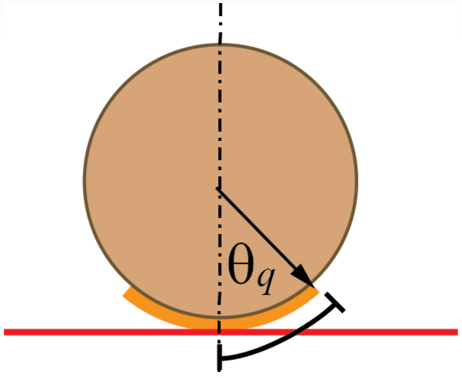
- Dielectric particle on a conducting plane
- Air as the exterior medium ($\varepsilon_E = 1$)
- Particle dielectric constant ε_p varied from 1 – 5
- Charging models: consider the same total charge amount



Uniform Charging



Top-pole Charging



Bottom-pole Charging

Method

- Boundary element method (BEM) applied to calculate electric field
- The BEM is suitable for highly nonuniform field cases
- 720 2nd-order elements, 1441 Nodes used for contour
- Boundary condition (of **E**) on the particle surfce

Without charge

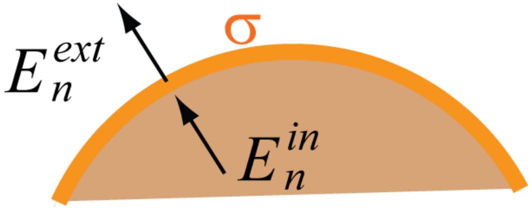
$$E_n^{ext} - \varepsilon_p E_n^{in} = 0$$

With charge density σ:

$$E_n^{ext} - \varepsilon_p E_n^{in} = \frac{\sigma}{\varepsilon_0}$$

- Force calculation

$$\mathbf{F}_a = \varepsilon_E \varepsilon_0 \int_S \left(\mathbf{E} E_n - \frac{1}{2} E^2 \mathbf{n} \right) \mathrm{d}S$$

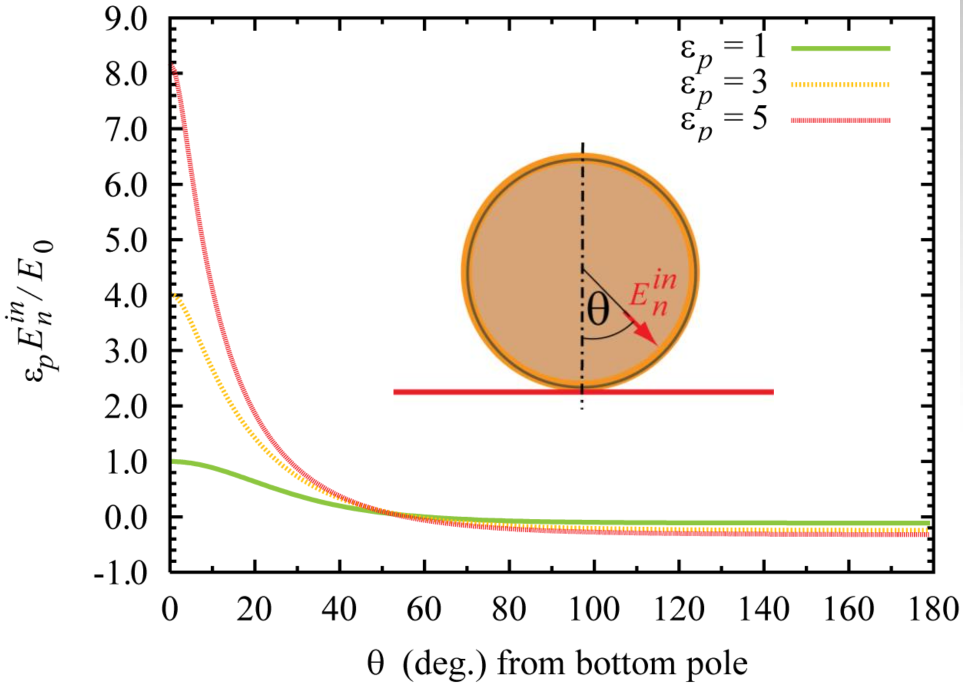


Effect of ε_p

- Consider uniform charging
- Peak electric field at the bottom pole
- Higher ε_p → Stronger field near the bottom pole
- On the exterior side

$$\frac{E_n^{ext}}{E_0} - \frac{\varepsilon_p E_n^{in}}{E_0} = 1$$

E_n^{ext} positive (outward) for all θ



σ/ε_0 = 1 V/m and $E_0 = \sigma/4\pi\varepsilon_0 R^2$

Effect of ε_p

- Expression of $\mathbf{F}_a$ of a charged particle on a grounded plane

$$F_a = \alpha \frac{Q^2}{4\pi\varepsilon_E\varepsilon_0(2R)^2}$$

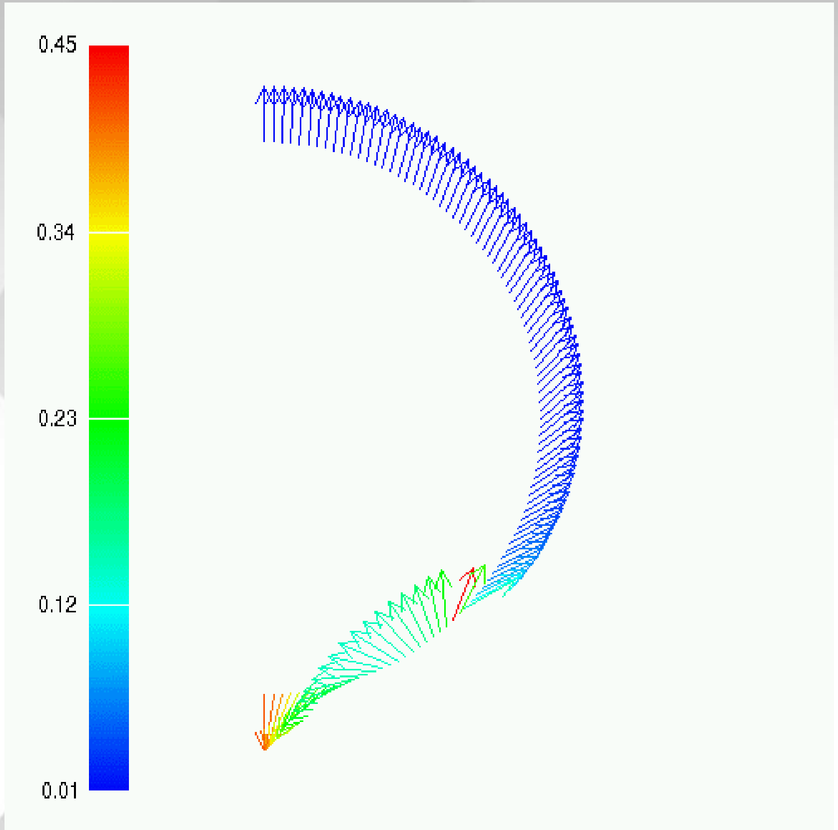
- α : force magnification factor
- D.A. Hays calculated α = 1.9 for ε_p = 4

α values obtained from BEM calculation

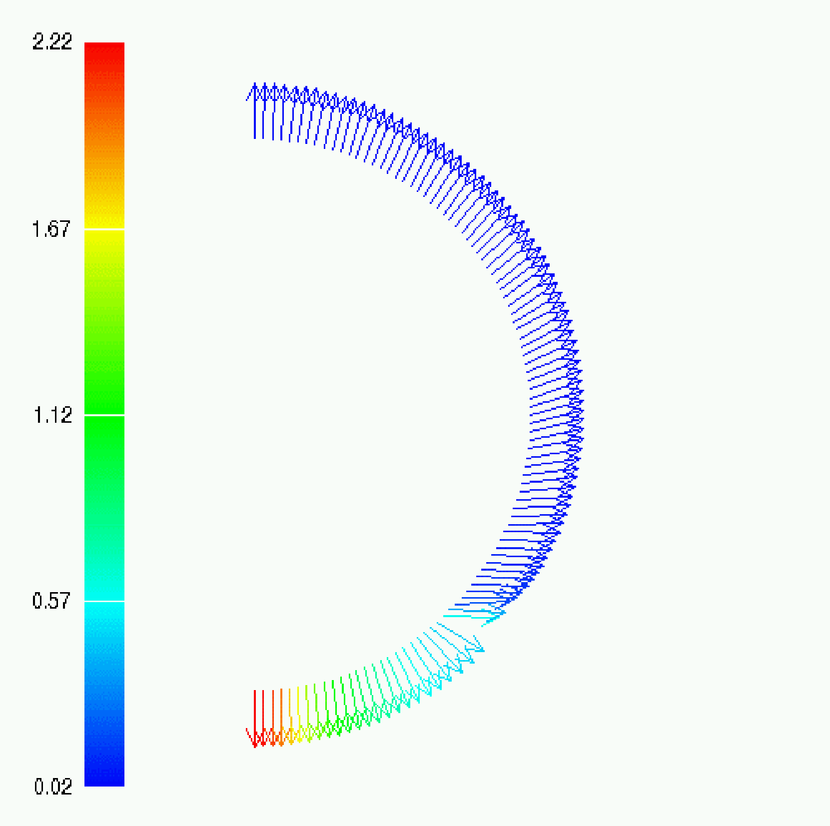
ε_p	α
1	1.0
2	1.29
3	1.61
4	1.95

Field Distr.: Partially Charged

In the Interior



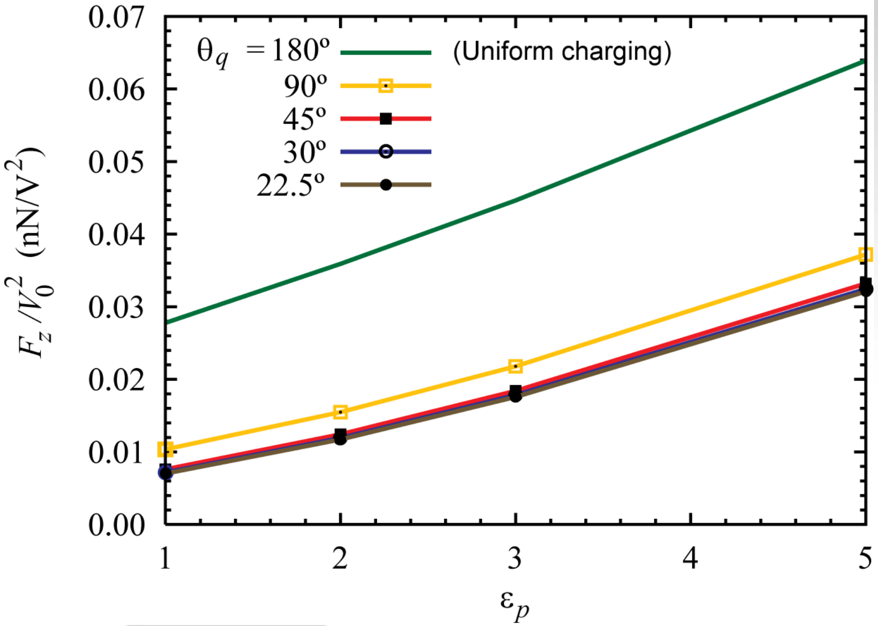
In the Exterior



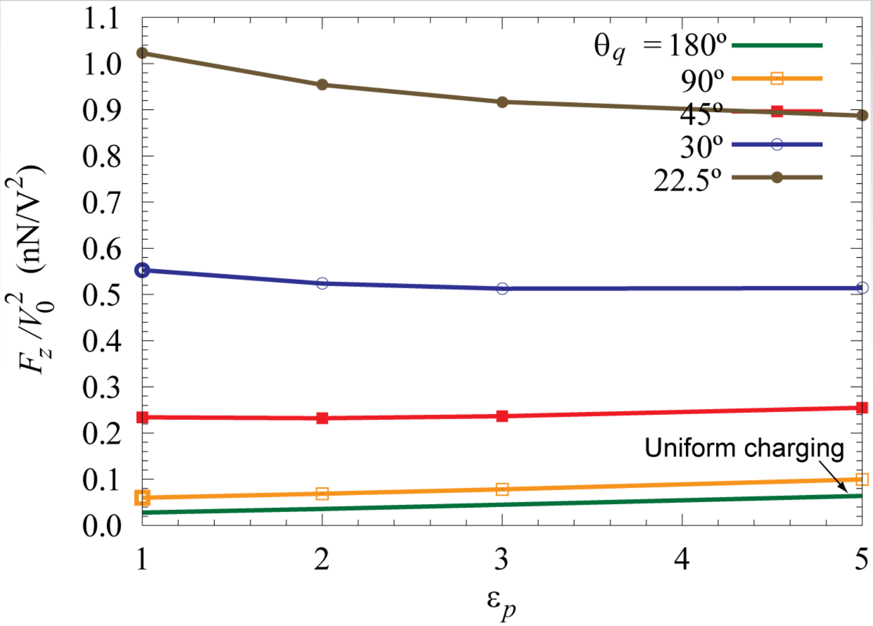
Bottom-pole charging with θ_q = 45°, ε_p = 3 and σ/ε_0 = 1 V/m

Adhesive Force Variation

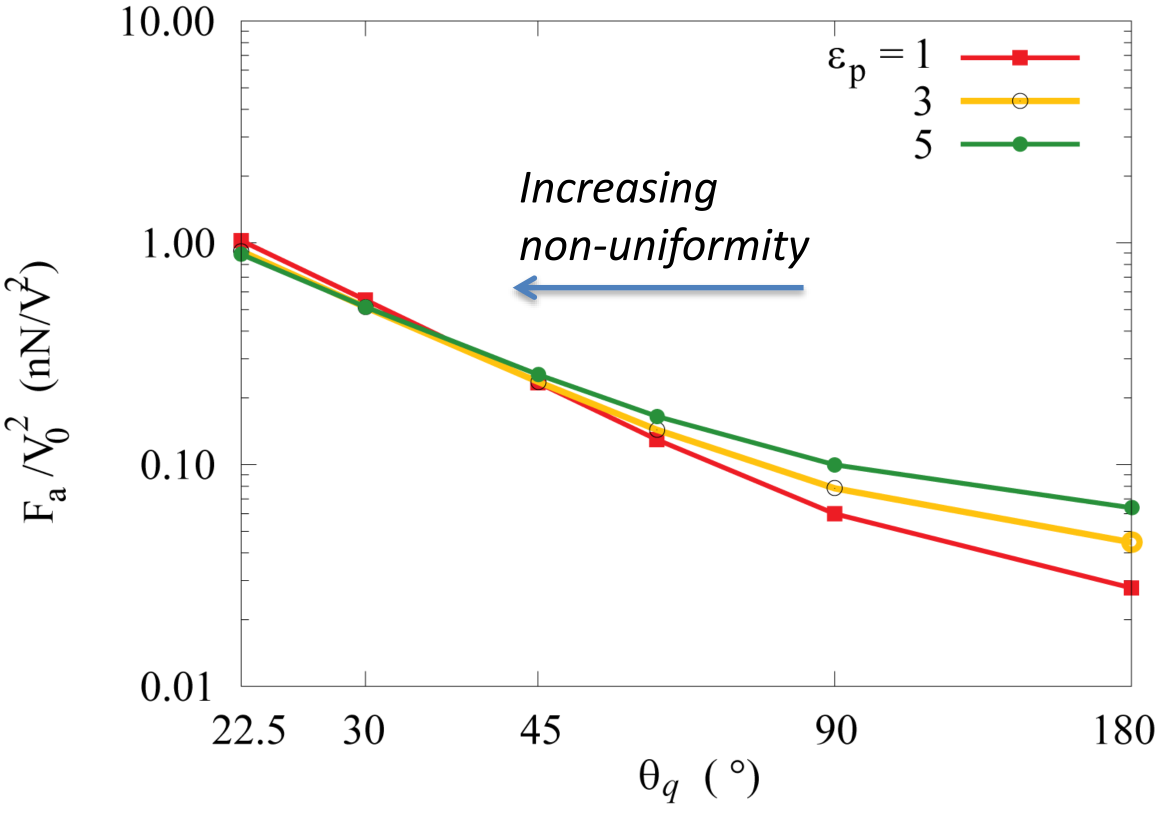
Top-Pole Distribution



Bottom-Pole Distribution



Force per V_0^2; $V_0 = Q/4\pi\varepsilon_0 R$



Variation of F_a with θ_q for bottom-pole charging

- For large θ_q or uniform, F_a increases with dielectric constant
- For small θ_q, F_a slightly decreases with dielectric constant

Comparison with Measured Data

- Toner particles (5.5 μm R) for laser printers
- For uniform charging with Q = 10 μC/g, measured $F \cong$ 100 nN*.
  - Simplest approximation gives 2.46 nN adhesion
  - Taking into account ε_p, 3.96 nN
  - Still one digit smaller than measured values
- Much stronger adhesion possible by nonuniform charging

Calculated results for 5-μm radius, 1000-kg/m^3 density particle and 10 μC/g total charge

- Q = 5.2 fC
- V_Q = 9.4 V

θ_q (°)	Adhesive Force, F_a (nN)		
	ε_p = 1	ε_p = 3	ε_p = 5
180	2.46	3.96	5.66
90	5.31	6.92	8.83
45	20.7	21.0	22.6
22.5	90.6	81.2	78.6

Conclusion

- We analyze electrostatic adhesion of a particle with partial charge distribution
- Numerical method is used as the tool to treat the discrete charge density
- Force becomes stronger when charges are at the bottom pole, but weaker when charges are at the top pole
- Effect of particle dielectric constant enhances adhesion in cases of uniform and slightly nonuniform
- Nonuniform charging can be the reason of discrepancy between analysis and measurement